Professional Standards for Teaching Mathematics
National Council for Teachers of Mathematics, NCTM 1991

A Vision for School Mathematics
Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers’ help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it.

The Standards
Six standards encompass the vision's core dimensions. These standards are grouped under four headings: Tasks, Discourse, Environment, and Analysis. All major arenas of teachers' work that are logically central to shaping what goes on in mathematics classes.

Tasks are the projects, questions, problems, constructions, applications, and exercises in which students engage. They provide the intellectual contexts for students' mathematical development.

Standard 1: Worthwhile Mathematical Tasks
The teacher of mathematics should pose tasks that are based on-
- sound and significant mathematics;
- knowledge of students' understandings, interests, and experiences;
- knowledge of the range of ways that diverse students learn mathematics;
and
- engage students' intellect;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;
- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to, and draw on, students' diverse background experiences and dispositions;
- promote the development of all students' dispositions to do mathematics.
Discourse refers to the ways of representing, thinking, talking, and agreeing and disagreeing that teachers and students use to engage in those tasks. The discourse embeds fundamental values about knowledge and authority. Its nature is reflected in what makes an answer right and what counts as legitimate mathematical activity, argue and thinking. Teachers, through the ways in which they orchestrate discourse, convey messages about whose knowledge and ways of thinking and knowing are valued, who is considered able to contribute and who has status in the group.

**Standard 2 - Teacher's Role in Discourse**
The teacher of mathematics should orchestrate discourse by-

- posing questions and tasks that elicit, engage, and challenge each student's thinking;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate

**Standard 3: Students' Role in Discourse**
The teacher of mathematics should promote classroom discourse in which students-

- listen to, respond to, and question the teacher and one another;
- use a variety of tools to reason, make connections, solve problems, and communicate;
- initiate problems and questions;
- make conjectures and present solutions;
- explore examples and counterexamples to investigate a conjecture;
- try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers;
- rely on mathematical evidence and argument to determine validity.

**Standard 4: Tools for Enhancing Discourse**
The teacher of mathematics, in order to enhance discourse, should encourage and accept the use of-

- computers, calculators, and other technology,
- concrete materials used as models;
- pictures, diagrams, tables, and graphs;
- invented and conventional terms and symbols;
- metaphors, analogies, and stories;
- written hypotheses, explanations, and arguments;
- oral presentations and dramatizations
**Environment** represents the setting for learning. It is the unique interplay of intellectual, social, and physical characteristics that shapes the ways of knowing and working that are encouraged and expected in the classroom. It is the context in which the tasks and discourse are embedded; it also refers to the use of materials and space.

**Standard 5: Learning Environment**
The teacher of mathematics should create a learning environment that fosters the development of each student's mathematical power by-

- providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems;
- using the physical space and materials in ways that facilitate students' learning of mathematics;
- providing a context that encourages the development of mathematical skill and proficiency;
- respecting and valuing students' ideas, ways of thinking, and mathematical dispositions;

and by consistently expecting and encouraging students to-

- work independently or collaboratively to make sense of mathematics;
- take intellectual risks by raising questions and formulating conjectures;
- display a sense of mathematical competence by validating and supporting ideas with mathematical argument.

**Analysis** is the systematic reflection in which teachers engage. It entails the ongoing monitoring of classroom life—how well the tasks, discourse, and environment foster the development of every student's mathematical literacy and power. Through this process, teachers examine relationships between what they and their students are doing and what students are learning.

**Standard 6: Analysis of Teaching and Learning**
The teacher of mathematics should engage in ongoing analysis of teaching and learning by-

- observing, listening to, and gathering other information about students to assess what they are learning;
- examining effects of the task, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions;
- in order to-

- ensure that every student is learning sound and significant mathematics and is developing a positive disposition toward mathematics;
- challenge and extend students' ideas;
- adapt or change activities while teaching
- make plans, both short- and long-range;
- describe and comment on each student's learning to parents and administrators, as well as to the students themselves.

In deciding how to present and elaborate the ideas underlying each of the six standards, we confronted two basic dilemmas. First, teaching is an integrated activity. Although we can analyze the practice of teaching into these four arenas of teachers’ work—tasks, discourse, environment, and analysis—they are in fact interwoven and interdependent. The quality of the classroom environment, for example, is both a function of and an influence on the classroom discourse. Alternatively, tasks are shaped by the discourse that surrounds them and the environment in which work takes place. Our second dilemma was that professional standards for mathematics teaching should represent values about what contributes to good practice without prescribing it. *Such standards should offer a vision, not a recipe.*
ASSUMPTIONS

The standards for teaching are based on four assumptions about the practice of mathematics teaching:

1. The goal of teaching mathematics is to help all students develop mathematical power. The Curriculum and Evaluation Standards for School Mathematics furnishes the basis for a curriculum in which mathematical reasoning, communication, problem solving, and connections are central. Teachers must help every student develop conceptual and procedural understandings of number, operations, geometry, measurement, statistics, probability, functions, and algebra and the connections among ideas. They must engage all students in formulating and solving a wide variety of problems, making conjectures and constructing arguments, validating solutions, and evaluating the reasonableness of mathematical claims. Along with all this, teachers must foster in students the disposition to use and engage in mathematics, an appreciation of its beauty and utility, and a tolerance for getting stuck or sidetracked. Teachers must help students realize that mathematical thinking involves dead ends and detours and encourage them to persevere when confronted with a puzzling problem and to develop the self-confidence and interest to do so.

2. WHAT students learn is fundamentally connected with HOW they learn it. Students' opportunities to learn mathematics are a function of the setting and the kinds of tasks and discourse in which they participate. What students learn—about particular concepts and procedures as well as about thinking mathematically—depends on the ways in which they engage in mathematical activity in their classrooms. Their dispositions toward mathematics are also shaped by such experiences. Consequently, the goal of developing students’ mathematical power requires careful attention to pedagogy as well as to curriculum.

3. All students can learn to think mathematically. The goals described in the Curriculum and Evaluation Standards for School Mathematics are goals for all students. Goals such as learning to make conjectures, to argue about mathematics using mathematical evidence, to formulate and solve problems—even perplexing ones—and to make sense of mathematical ideas are not just for some group thought to be "bright" or "mathematically able." Every student can—and should—learn to reason and solve problems, to make connections across a rich web of topics and experiences, and to communicate mathematical ideas. By "every student" we mean specifically:

- students who have been denied access in any way to educational opportunities as well as those who have not;
- students who are African American, Hispanic, American Indian, and other minorities as well as those who are considered to be part the majority;
- students who are female as well as those who are male;
- students who have not been successful as well as those who have been successful in school and in mathematics.

4. Teaching is a complex practice and hence not reducible to recipes or prescriptions. First of all, teaching mathematics draws on knowledge from several domains: knowledge of mathematics, of diverse learners, of how students learn mathematics, of the contexts of classroom, school and society. Such knowledge is general. However, teachers must also consider the particular, for teaching is context-specific. Theoretical knowledge about adolescent development, for instance, can only partly inform a decision about particular students learning a particular mathematical concept in a given context. Second, as teachers weave together knowledge from these different domains to decide how to respond to a student’s question, how to represent a particular mathematical idea, how long to pursue the discussion of a problem, or what task to use to engage students in a new topic, they often find themselves having to balance multiple goals and considerations. Making such decisions depends on a variety of factors that cannot be determined in the abstract or governed by rules of thumb.

Because teaching mathematics well is a complex endeavor, it cannot be reduced to a recipe for helping students learn. Instead, good teaching depends on a host of considerations and understandings. Good teaching demands that teachers reason about pedagogy in professionally defensible ways within the particular contexts of their own work. The standards for teaching mathematics are designed to help guide the processes of such reasoning, highlighting issues that are crucial in creating the kind of teaching practice that supports the learning goals of the Curriculum and Evaluation Standards for School Mathematics. This section circumscribes themes and values but does not, indeed, and could not prescribe "right" practice.